Multiple Indicator Stationary Time Series Models

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This article is intended to complement previous research (Sivo, 1997; Sivo & Willson, 1998, in press) by discussing the propriety and practical advantages of specifying multivariate time series models in the context of structural equation modeling for time series and longitudinal panel data. Three practical considerations motivated this article. Unlike Marsh (1993), Sivo and Willson (2000) did not offer multiple indicator (latent order) equivalents to their autoregressive (AR), moving average (MA), and autoregressive-moving average (ARMA) models. Moreover, such models have yet to be discussed, despite Marsh's (1993) advocacy for multiple indicator models in general. Further motivating multiple indicator extensions of the AR, MA, and ARMA equivalent models is the fact that longitudinal studies often collect data on more than 1 related variable per occasion. Such multiple indicator models capitalize on 1 of the chief analytical advantages of structural equation modeling in that measurement error may be estimated directly.

The use of structural equation modeling (SEM) to evaluate longitudinal panel data is becoming increasingly pervasive, with a wide variety of investigations focusing on either stationary or nonstationary multiwave models. Among the general class of nonstationary models specified to explain the dynamics within longitudinal data, growth curve models have received a fair amount of attention (see Willett & Sayer, 1996). Such models may be used to investigate what accounts for individual changes over time.

Conversely, stationary models are used to investigate why repeated observations of the same measure over time fail to correlate perfectly. Among stationary models specified to depict longitudinal patterns of consistency, simplex and quasi-simplex models historically have dominated the literature. Indeed, very few alternatives to the traditionally posited simplex and quasi-simplex models have been investigated

and made available. In reaction to the paucity of alternative models, Marsh (1993) extended the number of models to include the one-factor model (related to the classical true score model), its multiple indicator equivalent, and the multiple indicator equivalents of both the simplex and quasi-simplex models. Expanding yet further the number of stationary models available for panel data, Sivo (1997) and Sivo and Willson (in press) studied and recommended three time series equivalent models, namely autoregressive (AR), moving average (MA) and autoregressive-moving average (ARMA) models. Demand for the MA and ARMA models, in particular, was rooted in the widely reported finding of correlated errors in longitudinal panel data (e.g., Jöreskog, 1979, 1981; Jöreskog & Sörbom 1977, 1989; Marsh, 1993; Marsh & Grayson, 1994; Rogosa, 1979) and the virtual absence of such models specifically designed for panel data. Although in recent years, some researchers have demonstrated how to use SEM to fit time series models to time series data (e.g., Hershberger, Corneal, & Molenaar, 1994; Hershberger, Molenaar, & Corneal, 1996; van Buuren, 1997), few researchers have notably applied time series methodology to panel data in the same manner. A discussion of the use of multiple indicator time series models for both time series and longitudinal panel data is needed.

Three practical considerations motivated this article. Unlike Marsh (1993), Sivo and Willson (2000) did not offer multiple indicator (latent order) equivalents to their AR, MA, and ARMA models. Moreover, such models have yet to be discussed, despite Marsh's (1993) advocacy for multiple indicator models in general. Further motivating multiple indicator extensions of the AR, MA, and ARMA equivalent models is the fact that longitudinal studies often collect data on more than one related variable per occasion. Indeed, this condition does not ipso facto call for a multiple indicator model (e.g., the time series process for each manifest variable measured over multiple occasions might be retained although alternatively integrated together into one full model). Nonetheless, the availability of such models, once established, offers researchers alternative models for consideration. Arguably, time-dependent latent factors and errors have the potential to evidence stochastic effects similar to those found among longitudinally assessed manifest variables. In fact, Bollen (1989) indicated that although it is traditionally assumed that latent errors (ζ) are nonautocorrelated, corrections for autocorrelated latent errors are common among econometric models and rarely studied for latent variable structural models. Given the widely recognized feasibility of autocorrelated latent factor scores, as represented by the quasi-simplex model (see Jöreskog and Sörbom, 1989), its second-order extension (see Marsh, 1993), or an instance of the dynamic shock-error model (Maravall & Aigner, 1977), it is surprising that no counterpart for autocorrelated latent factor errors exists. If autocorrelation among factor scores has been found to exist in longitudinal panel data, the possibility of autocorrelated latent errors should be considered as a counterpart. Furthermore, multiple indicator models capitalize on one of the chief analytical advantages of SEM in that measurement error may be estimated directly. This feature of SEM has the potential of greatly improving on classical time series modeling.

This article was written to expand the base of available stationary time series models for time series and panel data by presenting the specification of multiple indicator equivalents within the context of SEM. A general review of time series models is therefore warranted.

Time Series Models

Autocorrelation among serially observed scores (i.e., times series data) is a problematic condition that potentially biases parameter estimation, although it may be controlled through explicit modeling. Stationary times series data may be modeled for two stochastic processes: AR and MA (Box & Jenkins, 1976). AR models represent the most recent observation in a series as a function of previous observations within the same series. The most general univariate case is represented by

$$y_t = \phi_1 \, y_{t-1} + \phi_2 \, y_{t-2} + \dots + \phi_p \, y_{t-p} + \varepsilon_t$$

where $t = 1$ to T occasions, y_t denotes an observed score taken on some occasion (t) deviated from the original level y_0 of the series, ε denotes error associated with a given occasion (t), and $\phi \, (-1 < \phi < 1)$ denotes a covariance among temporally ordered scores at some lag (e.g., $t-1$ = a lag of 1, $t-2$ = a lag of 2). The autocorrelation function of an AR process has the characteristic of tapering off exponentially after the lag of the process. The multivariate counterpart of this general case is

$$\mathbf{y}_t = \Phi_1 \, \mathbf{y}_{t-1} + \Phi_2 \, \mathbf{y}_{t-2} + \dots + \Phi_p \, \mathbf{y}_{t-p} + \varepsilon_t$$

where the parameters are contained within the Φ matrixes. Following from the general univariate case, an AR model with a lag one relationship (i.e., AR1) is represented by

$$y_t = \phi_1 \, y_{t-1} + \varepsilon_t$$

and has the following multivariate counterpart

$$\mathbf{y}_t = \Phi_1 \, \mathbf{y}_{t-1} + \varepsilon_t$$

(see Appendix). The multivariate AR1 model is a restricted form of the simplex model (Willson, 1995).

Unlike AR models, MA models represent the most recent observation in a series as a function of autocorrelated errors among earlier observations. The most general univariate case is represented by

$$y_t = \varepsilon_t + \theta_1 \, \varepsilon_{t-1} + \theta_2 \, \varepsilon_{t-2} + \dots + \theta_q \, \varepsilon_{t-q}$$

where $t = 1$ to T occasions, y_t denotes an observed score taken on some occasion (t) deviated from the original level y_0 of the series, ε denotes error associated with a given occasion (t), and $\theta \, (-1 < \theta < 1)$ denotes a covariance among errors at some lag (e.g., $t-1$ = a lag of 1, $t-2$ = a lag of 2). By extension, the multivariate form of this model is

$$\mathbf{y}_t = \varepsilon_t + \theta_1 \, \varepsilon_{t-1} + \theta_2 \, \varepsilon_{t-2} + \dots + \theta_q \, \varepsilon_{t-q}$$

where the parameters are contained within the θ matrixes. Following from the general univariate case, an MA model with a lag one relationship (i.e., MA1) is represented by

$$y_t = \varepsilon_t + \varepsilon_1 \varepsilon_{t-1}$$

with the multivariate counterpart being

$$\mathbf{y}_t = \varepsilon_t + \theta_1 \varepsilon_{t-1}$$

(see Appendix). An MA1 model would have the error for the first occasion correlate with the second occasion error, and the second occasion error correlate with the third occasion error. However, the first occasion error would not be correlated with the third occasion error. This is possible when a unique component is introduced on each occasion, a component that covaries with a subsequent error but is independent of the previous error. Each unique component, jointly with the previous error, codetermines the following error in the series. The net effect of an MA1 process is that the autocorrelation function cuts off immediately after lag 1. Put simply, all error covariances beyond the first lag will be zero. Only the errors on temporally adjacent occasions possess a nonzero covariance and constitute the MA1 lag.

When both AR and MA processes are present in the same data, an ARMA model may best represent the variation in the data. The univariate form of the most general case of the ARMA model is

$$y_t = \phi_1 y_{t-1} + \phi_2 y_{t-2} + \ldots + \phi_p y_{t-p} + \theta_1 \varepsilon_{t-1} + \theta_2 \varepsilon_{t-2} + \ldots + \theta_q \varepsilon_{t-q} + \varepsilon_t$$

and the multivariate form is

$$\mathbf{y}_t = \Phi_1 \mathbf{y}_{t-1} + \Phi_2 \mathbf{y}_{t-2} + \ldots + \Phi_p \mathbf{y}_{t-p} + \theta_1 \varepsilon_{t-1} + \theta_2 \varepsilon_{t-2} + \ldots + \theta_q \varepsilon_{t-q} + \varepsilon_t$$

(see Appendix). The ARMA model with a lag one relationship for both its AR and MA processes is represented by

$$y_t = \phi_1 y_{t-1} + \varepsilon_t + \theta_1 \varepsilon_{t-1}$$

and its multivariate form is

$$\mathbf{y}_t = \mathbf{\Phi_1} \mathbf{y_{t-1}} + \varepsilon_t + \theta_1 \varepsilon_{t-1}$$

Harvey (1981) unequivocally pointed out that a multivariate time series model is founded on a priori assumptions inasmuch as it suggests that the variables under study are determined jointly. From Harvey's perspective, the idea of modeling individual trajectories in addition to the multivariate trajectory would amount to a loss in predictive efficiency, especially when the multivariate time series model is specified correctly. This point cannot be understated and has great relevance to the contextual scheme of this article because the approach to be advanced will not consider or advocate the use of univariate time series models for individual trajectories. A limited information approach, modeling each univariate process, is only strategically meritorious when uncertainty exists about the model as a whole (Harvey, 1981). In such a case, SEM would be un-

warranted, given that this approach assumes that an asserted model is grounded theoretically in the first place.

On a more practical note, it should be pointed out that, as Hershberger et al. (1996) discussed, SAS IML may be used to create the autocovariance and block toeplitz matrixes necessary so that the multiple indicator SEM specified time series models may be fit to the multivariate time series covariance data.

The Relation Between Time Series and Longitudinal Panel Designs

A clear connection has been delineated between time series designs and longitudinal panel designs in which the same sample of cases is observed on multiple occasions. Rogosa (1979) indicated that "[longitudinal] panel designs are a combination of time-series and cross-sectional, with measurements obtained on a cross-section (wave) at each time point" (p. 275). According to Fredericksen and Rotondo (1979), "When a time series model is employed [with] … suitable techniques for parameter estimation and hypothesis testing, the result is a powerful methodology for the conduct of longitudinal research" (p. 112). To be sure, longitudinal data may be treated similarly to time series, when (a) the same group of individuals over occasions are measured (i.e., panel study), (b) the occasions for repeated measurements are equidistant in time, and (c) enough measurement occasions over time are included. Regarding the latter condition, Box and Jenkins (1976) indicated that when modeling individual trajectories, at least 50 observations are needed for unambiguous model identification. Although, if a priori models can be assumed (e.g., AR1 or MA1), far fewer observations are needed. Sivo and Willson (1998) indicated that as few as four time points are reasonable for fitting AR1 or MA1 models to large-sample panel data, wherein individual performances may be considered replications and cross time covariances are thereby more stable. Five or six occasions at minimum are recommended when testing ARMA (1,1) models.

Fitting Time Series Models to Longitudinal Panel Data

Sivo and Willson (2000) defined the AR1 model for longitudinal panel data in a manner consistent with time series methodology.

$$\mathbf{y_t} = \beta_{21} \, \mathbf{y_{t-1}} + \varepsilon_t, \, t = 1 \text{ to } T \text{ occasions}$$

The equations that define the multiple indicator AR1 model will resemble the single indicator AR1 model, though the AR process is represented among latent factors instead of manifest variables.

$$\mathbf{y_i} = \lambda_{21} \, \eta_t + \varepsilon_i, \, i = 1 \text{ to } p \text{ variables}$$

$$\eta_t = \beta_{21} \, \eta_{t-1} + \zeta_t, \, t = 1 \text{ to } T \text{ occasions}$$

When specifying a model to represent an autoregressive process among six latent factors, the matrix equation for the factor relations would be

$$
\begin{bmatrix} \eta_1 \\ \eta_2 \\ \eta_3 \\ \eta_4 \\ \eta_5 \\ \eta_6 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ \beta_{21} & 0 & 0 & 0 & 0 & 0 \\ 0 & \beta_{21} & 0 & 0 & 0 & 0 \\ 0 & 0 & \beta_{21} & 0 & 0 & 0 \\ 0 & 0 & 0 & \beta_{21} & 0 & 0 \\ 0 & 0 & 0 & 0 & \beta_{21} & 0 \end{bmatrix} \begin{bmatrix} \eta_1 \\ \eta_2 \\ \eta_3 \\ \eta_4 \\ \eta_5 \\ \eta_6 \end{bmatrix} + \begin{bmatrix} \zeta_1 \\ \zeta_2 \\ \zeta_3 \\ \zeta_4 \\ \zeta_5 \\ \zeta_6 \end{bmatrix}
$$

(see Appendix). This model resembles the quasi-simplex model (see Jöreskog & Sörbom, 1989, p. 182), although two differences exist: (a) Each factor determines multiple indicators, and (b) the betas are constrained to equal the first beta in the series (see Figure 1).

The MA1 model specified for longitudinal panel data also assumes a form consistent with time series methodology.

$$y_t = \theta_{21}\, \varepsilon_{t-1} + \varepsilon_t, \ t = 1 \text{ to } T \text{ occasions}$$

Because the theta epsilon matrix is symmetrical and the MA1 model requires an asymmetrical specification among the lag 1 errors, it is useful to define the model in a manner similar to van Buuren's (1997) approach. Van Buuren discussed the use of SEM to estimate univariate time series, and so his general model for a univariate MA process represents the relation between the original series and each of the hypothesized lags for the series. To specify the asymmetrical relation between each lag and the origin, van Buuren defined the errors as factors that are related to the order of some lag. His MA1 model is specified by the following CALIS equations:

$$v_0 = f0 + theta1\ f1$$

$$v_1 = f1 + theta1\ f2$$

which may be re-expressed as

$$y_0 = \lambda_1\, \eta_1 + \eta_0$$

$$y_1 = \lambda_1\, \eta_2 + \eta_1$$

His use of the lambda matrix allows him to make the asymmetrical specification of a lagged relation among the errors, a relation that could not have been asymmetrically specified within a theta epsilon matrix. Similarly, the MA1 longitudinal model may be expressed as

$$y_t = \lambda_{21}\, \eta_{t-1} + \eta_t, \ t = 1 \text{ to } T \text{ occasions}$$
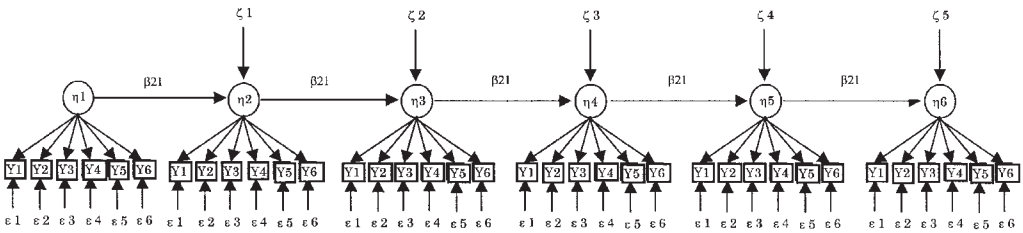
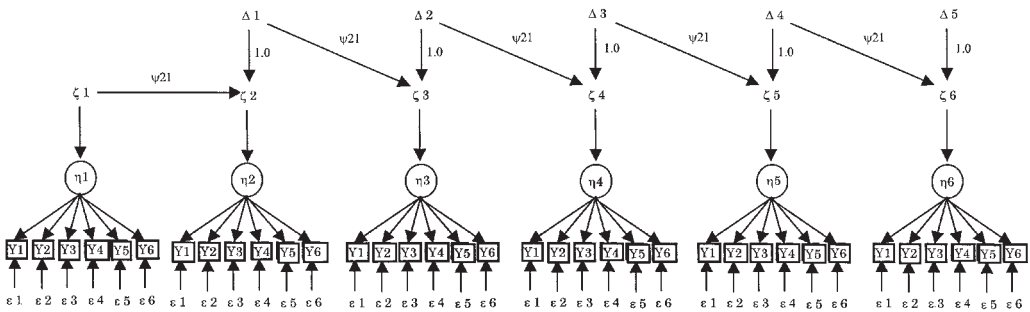FIGURE 1   Multiple indicator autoregressive (AR1) model.

FIGURE 2   Multiple indicator moving average (MA1) model.

In this case, errors across time may be said to correlate at some lag. Again, this retains the original form of the MA1 time series model discussed previously:

$$\mathbf{y}_t = \varepsilon_t + \theta_1 \, \varepsilon_{t-1}$$

The equations that define the multiple-indicator MA1 model will resemble the single-indicator MA1 model, although the MA process is represented among latent factors instead of manifest variables (see Appendix).

$$\mathbf{y_i} = \lambda_{\mathbf{it}} \, \eta_{\mathbf{t}} + \varepsilon_{\mathbf{i}}, \, i = 1 \text{ to } p \text{ variables}$$

$$\eta_{\mathbf{t}} = \psi_{\mathbf{21}} \, \zeta_{\mathbf{t-1}} + \zeta_{\mathbf{t}}, \, t = 1 \text{ to } T \text{ occasions}$$

Re-expressing the multiple indicator MA process in terms of factors yields

$$\eta_{\mathbf{t}} = \gamma_{\mathbf{21}} \, \xi_{\mathbf{t-1}} + \xi_t, \, t = 1 \text{ to } T \text{ occasions}$$

Refer to Figure 2 for a diagram of this model. Although the pure multiple indicator MA model, as specified, is theoretically accurate, it may not seem to make any

practical sense. If a set of manifest variables is determined by a common factor on each of several occasions, it is challenging to conceive of how an MA process alone could explain the relatedness among the first-order factors. If an MA process alone is responsible for the relation among the factors, then how could each factor determine the conceptually related manifest variables in practice? Consequently, identifying an MA process present within some lag of the latent errors may be more likely under the condition that the latent factors are otherwise related as well. This is not too troubling when it is recalled that the MA process is but a nuisance condition. As such, one would want to model explicitly an MA1 process as a form of control to remove the bias that the process would otherwise introduce into parameter estimates for a given model, say, a one-factor model.

$$\mathbf{y_i} = \lambda_{it}\, \eta_t + \varepsilon_i, \; i = 1 \text{ to } p \text{ variables}$$

$$\eta_t = \gamma_{t1}\, \xi_1 + \psi_{21}\, \zeta_{t-1} + \zeta_t, \; t = 1 \text{ to } T \text{ occasions}$$

which may also be expressed as

$$\eta_t = \gamma_{t\,T+1}\, \xi_{T+1} + \gamma_{21}\, \xi_{t-1} + \xi_t, \; t = 1 \text{ to } T \text{ occasions}$$

If the same variables were measured on six occasions, the multiple indicator MA1 process thought to theoretically relate temporally adjacent latent errors would be specified by the following matrix equation:

$$
\begin{bmatrix} \eta_1 \\ \eta_2 \\ \eta_3 \\ \eta_4 \\ \eta_5 \\ \eta_6 \end{bmatrix}
=
\begin{bmatrix}
\gamma_{17} & 1.0 & 0 & 0 & 0 & 0 & 0 \\
\gamma_{27} & \gamma_{28} & 1.0 & 0 & 0 & 0 & 0 \\
\gamma_{37} & 0 & \gamma_{28} & 1.0 & 0 & 0 & 0 \\
\gamma_{47} & 0 & 0 & \gamma_{28} & 1.0 & 0 & 0 \\
\gamma_{57} & 0 & 0 & 0 & \gamma_{28} & 1.0 & 0 \\
\gamma_{67} & 0 & 0 & 0 & 0 & \gamma_{28} & 1.0
\end{bmatrix}
\begin{bmatrix} \eta_7 \\ \eta_8 \\ \eta_9 \\ \eta_{10} \\ \eta_{11} \\ \eta_{12} \\ \eta_{13} \end{bmatrix}
$$

Note that the gammas for the second order-factor are also specified in Γ (see Figure 3). Although extraneous to the intent of the present investigation, this point is demonstrated as one practical possibility.

The ARMA (1,1) model specified for longitudinal panel data is expressed by the following equation:

$$\mathbf{y_t} = \beta_1\, \mathbf{y_{t-1}} + \varepsilon_t - \theta_1\, \varepsilon_{t-1}, \; t = 1 \text{ to } T \text{ occasions}$$

whereas the multiple indicator equivalent is defined as

$$\mathbf{y_i} = \lambda_{21}\, \eta_t + \varepsilon_i, \; i = 1 \text{ to } p \text{ variables}$$

$$\eta_t = \beta_{21}\, \eta_{t-1} + \zeta_t + \psi_{21}\, \zeta_{t-1}, \; t = 1 \text{ to } T \text{ occasions}$$

Re-expressing the multiple indicator MA process in terms of factors yields
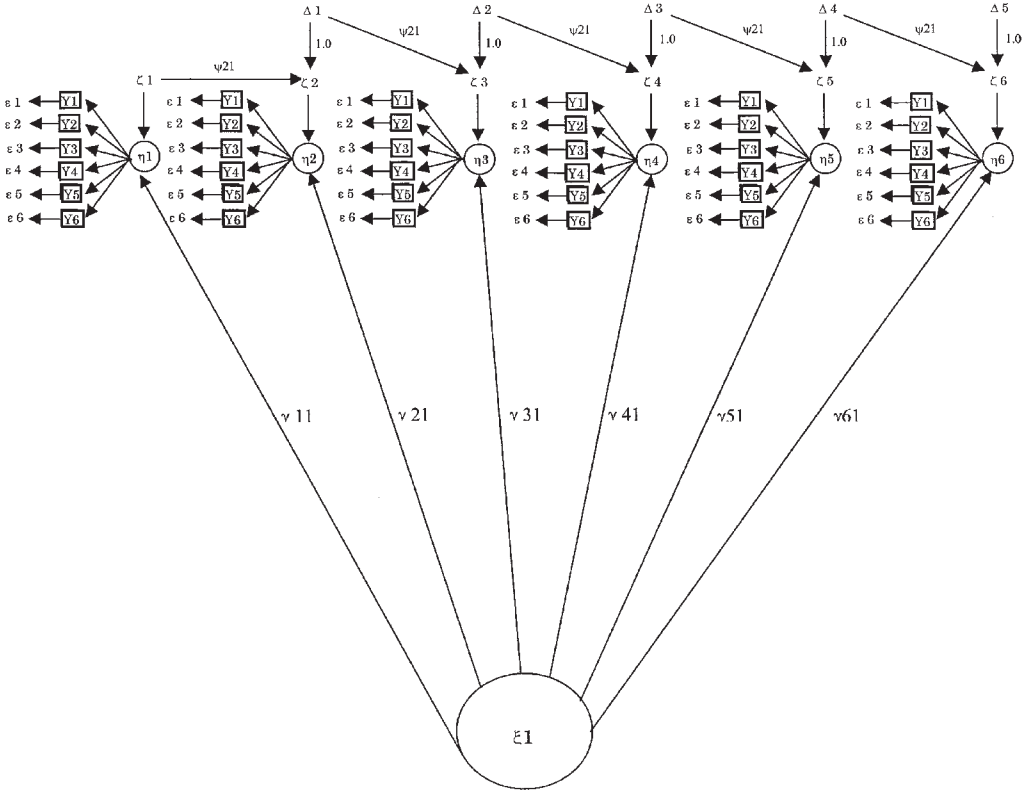
FIGURE 3   Multiple indicator moving average (MA1) model with second-order factor.

$$\eta_t = \beta_{21}\,\eta_{t-1} + \xi_t + \gamma_{21}\,\xi_{t-1}, \; t = 1 \text{ to } T \text{ occasions}$$

which for a six-occasion situation may be expressed by the following matrix equation:

$$
\begin{bmatrix} \eta_1 \\ \eta_2 \\ \eta_3 \\ \eta_4 \\ \eta_5 \\ \eta_6 \end{bmatrix}
=
\begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 \\
\beta_{21} & 0 & 0 & 0 & 0 & 0 \\
0 & \beta_{21} & 0 & 0 & 0 & 0 \\
0 & 0 & \beta_{21} & 0 & 0 & 0 \\
0 & 0 & 0 & \beta_{21} & 0 & 0 \\
0 & 0 & 0 & 0 & \beta_{21} & 0
\end{bmatrix}
\begin{bmatrix} \eta_1 \\ \eta_2 \\ \eta_3 \\ \eta_4 \\ \eta_5 \\ \eta_6 \end{bmatrix}
+
\begin{bmatrix}
1.0 & 0 & 0 & 0 & 0 & 0 \\
\gamma_{28} & 1.0 & 0 & 0 & 0 & 0 \\
0 & \gamma_{28} & 1.0 & 0 & 0 & 0 \\
0 & 0 & \gamma_{28} & 1.0 & 0 & 0 \\
0 & 0 & 0 & \gamma_{28} & 1.0 & 0 \\
0 & 0 & 0 & 0 & \gamma_{28} & 1.0
\end{bmatrix}
\begin{bmatrix} \eta_7 \\ \eta_8 \\ \eta_9 \\ \eta_{10} \\ \eta_{11} \\ \eta_{12} \end{bmatrix}
$$

(see Appendix).
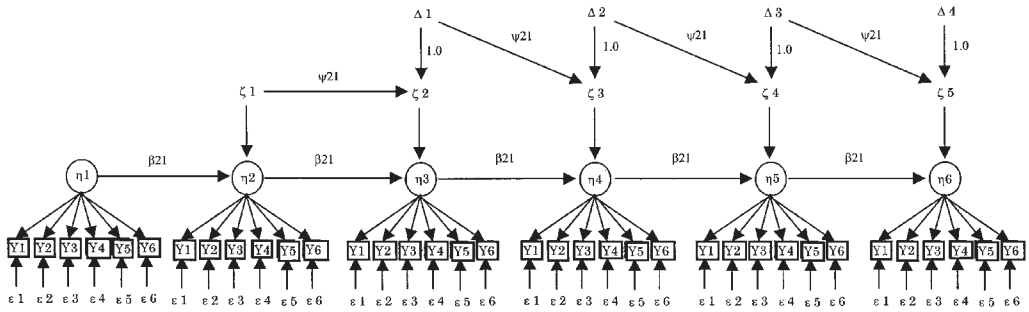Refer to Figure 4 for a diagram of this model.

FIGURE 4   Multiple indicator autoregressive-Moving Average (ARMA 1, 1) model.

CONCLUSION

This article described why fitting multiple indicator time series models to time series and panel data within the context of structural equation modeling is proper, practicable, and useful. With regard to modeling time series data, the multiple indicator model specification improves on classical time series analysis in that it allows measurement errors to be directly modeled, a key feature that has made SEM more broadly a strategically useful analytical approach. With regard to modeling panel data, the multiple indicator model specification improves on the work of Sivo and Willson (2000) in that it extends the base of plausible stationary models recommended for certain types of panel data. Finally, this article demonstrates how multiple indicator time series models may be specified by using SAS's PROC CALIS.

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APPENDIX

Autoregressive (AR) Program

```
PROC CALIS DATA=P COV ALL;
              TITLE 'AR MODEL: MULTIPLE INDICATOR FORM';
LINEQS
    I1T1 = LY11 F1+E1, I2T1 = LY21 F1+E2, I3T1 = LY31 F1+E3,
    I4T1 = LY41 F1+E4, I5T1 = LY51 F1+E5, I6T1 = LY61 F1+E6,
```

```
  I1T2 = LY11 F2+E7, I2T2 = LY21 F2+E8, I3T2 = LY31 F2+E9,
  I4T2 = LY41 F2+E10, I5T2 = LY51 F2+E11, I6T2 = LY61 F2+E12,
  I1T3 = LY11 F3+E13, I2T3 = LY21 F3+E14, I3T3 = LY31 F3+E15,
  I4T3 = LY41 F3+E16, I5T3 = LY51 F3+E17, I6T3 = LY61 F3+E18,
  I1T4 = LY11 F4+E19, I2T4 = LY21 F4+E20, I3T4 = LY31 F4+E21,
  I4T4 = LY41 F4+E22, I5T4 = LY51 F4+E23, I6T4 = LY61 F4+E24,
  I1T5 = LY11 F5+E25, I2T5 = LY21 F5+E26, I3T5 = LY31 F5+E27,
  I4T5 = LY41 F5+E28, I5T5 = LY51 F5+E29, I6T5 = LY61 F5+E30,
  I1T6 = LY11 F6+E31, I2T6 = LY21 F6+E32, I3T6 = LY31 F6+E33,
  I4T6 = LY41 F6+E34, I5T6 = LY51 F6+E35, I6T6 = LY61 F6+E36,
    F2 = ARlag1 F1+D1,
    F3 = ARlag1 F2+D2,
    F4 = ARlag1 F3+D3,
    F5 = ARlag1 F4+D4,
    F6 = ARlag1 F5+D5;
STD
  e1-e36=ManErr1-ManErr36, D1-D5=ZetaErr1-ZetaErr5, F1=1;
VAR
  I1T1 I2T1 I3T1 I4T1 I5T1 I6T1
  I1T2 I2T2 I3T2 I4T2 I5T2 I6T2
  I1T3 I2T3 I3T3 I4T3 I5T3 I6T3
  I1T4 I2T4 I3T4 I4T4 I5T4 I6T4
  I1T5 I2T5 I3T5 I4T5 I5T5 I6T5
  I1T6 I2T6 I3T6 I4T6 I5T6 I6T6;


Moving-Average (MA) Program

PROC CALIS DATA=P COV ALL;
            TITLE 'MA MODEL: MULTIPLE INDICATOR FORM';
LINEQS
  I1T1 = LY11 F1+E1, I2T1 = LY21 F1+E2, I3T1 = LY31 F1+E3,
  I4T1 = LY41 F1+E4, I5T1 = LY51 F1+E5, I6T1 = LY61 F1+E6,
  I1T2 = LY12 F2+E7, I2T2 = LY22 F2+E8, I3T2 = LY32 F2+E9,
  I4T2 = LY42 F2+E10, I5T2 = LY52 F2+E11, I6T2 = LY62 F2+E12,
  I1T3 = LY13 F3+E13, I2T3 = LY23 F3+E14, I3T3 = LY33 F3+E15,
  I4T3 = LY43 F3+E16, I5T3 = LY53 F3+E17, I6T3 = LY63 F3+E18,
  I1T4 = LY14 F4+E19, I2T4 = LY24 F4+E20, I3T4 = LY34 F4+E21,
  I4T4 = LY44 F4+E22, I5T4 = LY54 F4+E23, I6T4 = LY64 F4+E24,
  I1T5 = LY15 F5+E25, I2T5 = LY25 F5+E26, I3T5 = LY35 F5+E27,
  I4T5 = LY45 F5+E28, I5T5 = LY55 F5+E29, I6T5 = LY65 F5+E30,
  I1T6 = LY16 F6+E31, I2T6 = LY26 F6+E32, I3T6 = LY36 F6+E33,
  I4T6 = LY46 F6+E34, I5T6 = LY56 F6+E35, I6T6 = LY66 F6+E36,
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     F1 = F7,
     F2 = MAlag1 F7 + F8,
     F3 = MAlag1 F8 + F9,
     F4 = MAlag1 F9 + F10,
     F5 = MAlag1 F10 + F11,
     F6 = MAlag1 F11 + F12;
STD
  e1-e36=ManErr1-ManErr36, F7-F12=6*1;
VAR
  I1T1 I2T1 I3T1 I4T1 I5T1 I6T1
  I1T2 I2T2 I3T2 I4T2 I5T2 I6T2
  I1T3 I2T3 I3T3 I4T3 I5T3 I6T3
  I1T4 I2T4 I3T4 I4T4 I5T4 I6T4
  I1T5 I2T5 I3T5 I4T5 I5T5 I6T5
  I1T6 I2T6 I3T6 I4T6 I5T6 I6T6;
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Autoregressive Moving-Average (ARMA) Program

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PROC CALIS DATA=P COV ALL;
              TITLE 'ARMA MODEL: MULTIPLE INDICATOR FORM';
LINEQS
  I1T1 = LY11 F1+E1, I2T1 = LY21 F1+E2, I3T1 = LY31 F1+E3,
  I4T1 = LY41 F1+E4, I5T1 = LY51 F1+E5, I6T1 = LY61 F1+E6,
  I1T2 = LY11 F2+E7, I2T2 = LY21 F2+E8, I3T2 = LY31 F2+E9,
  I4T2 = LY41 F2+E10, I5T2 = LY51 F2+E11, I6T2 = LY61 F2+E12,
  I1T3 = LY11 F3+E13, I2T3 = LY21 F3+E14, I3T3 = LY31 F3+E15,
  I4T3 = LY41 F3+E16, I5T3 = LY51 F3+E17, I6T3 = LY61 F3+E18,
  I1T4 = LY11 F4+E19, I2T4 = LY21 F4+E20, I3T4 = LY31 F4+E21,
  I4T4 = LY41 F4+E22, I5T4 = LY51 F4+E23, I6T4 = LY61 F4+E24,
  I1T5 = LY11 F5+E25, I2T5 = LY21 F5+E26, I3T5 = LY31 F5+E27,
  I4T5 = LY41 F5+E28, I5T5 = LY51 F5+E29, I6T5 = LY61 F5+E30,
  I1T6 = LY11 F6+E31, I2T6 = LY21 F6+E32, I3T6 = LY31 F6+E33,
  I4T6 = LY41 F6+E34, I5T6 = LY51 F6+E35, I6T6 = LY61 F6+E36,
     F1 = F7,
     F2 = ARLag1 F1 + F8 + MAlag1 F7 ,
     F3 = ARLag1 F2 + F9 + MAlag1 F8 ,
     F4 = ARLag1 F3 + F10 + MAlag1 F9 ,
     F5 = ARLag1 F4 + F11 + MAlag1 F10,
     F6 = ARLag1 F5 + F12 + MAlag1 F11;
STD
  e1-e36=ManErr1-ManErr36, F7-F12=6*1;
VAR
```

I1T1 I2T1 I3T1 I4T1 I5T1 I6T1
I1T2 I2T2 I3T2 I4T2 I5T2 I6T2
I1T3 I2T3 I3T3 I4T3 I5T3 I6T3
I1T4 I2T4 I3T4 I4T4 I5T4 I6T4
I1T5 I2T5 I3T5 I4T5 I5T5 I6T5
I1T6 I2T6 I3T6 I4T6 I5T6 I6T6;